



JENN

Training and Consultancy

The path to enlightened education

VAK: WISKUNDE

GRAAD 12

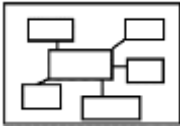





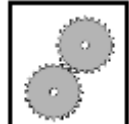

2025 LENTESKOOL

OPLOSSINGSHANDLEIDING

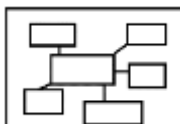





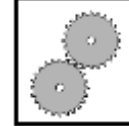

ONDERWERPE

- 1. FINANSIES, GROEI EN VERVAL**
- 2. STATISTIEK EN REGRESSIE**
- 3. ANALITIESE MEETKUNDE**

IKON BESKRYWING

			
BREINKAART	EKSAMEN RIGLYN	INHOUD	AKTIWITEITE
			
BIBLIOGRAFIE	TERMINOLOGIE	UITGEWERKTE VOORBEELDE	STAPPE

ICON DESCRIPTION

			
MIND MAP	EXAMINATION GUIDELINE	CONTENTS	ACTIVITIES
			
BIBLIOGRAPHY	TERMINOLOGY	WORKED EXAMPLES	STEPS



INHOUDSOPGAWE

BLADSY

AFDELING 1: Finansies, Groei en Verval Oplossings	4 – 6
AFDELING 2: Statistiek en Regressie Oplossings	7 – 13
AFDELING 3: Analitiese Meetkunde Oplossings	14 – 28

Finansies, Groei en Verval

Mei/Junie 2024

QUESTION 7/VRAAG 7

7.1	$A = P(1-i)^n$ $8\,337,75 = 13\,000(1-i)^6$ $i = 7,14\%$	✓ substitution in correct formula ✓✓ answer (3)
7.2	$F = \frac{x[(1+i)^n - 1]}{i}$ $80\,000 = \frac{x\left[\left(1 + \frac{8,6}{1200}\right)^{36} - 1\right]}{\frac{8,6}{1200}}$ $x = R1\,955,78$ <p>Thandi's total = $1955,78 \times 36 = R\,70\,408,08$ Eric's total = $1402,31 \times 48 = R\,67\,310,88$ Difference = $70\,409,08 - 67\,310,88$ = $R3\,097,20$</p>	✓ i ✓ substitution into correct formula ✓ answer ✓ answer (4)
7.3	$A = P(1+i)^n$ $A = 225\,000\left(1 + \frac{0,09}{12}\right)^3$ $A = R\,230\,100,5637\dots$ $225\,000\left(1 + \frac{0,09}{12}\right)^3 = \frac{5\,500\left[1 - \left(1 + \frac{0,09}{12}\right)^{-n}\right]}{\frac{0,09}{12}}$ $0,3137734959\dots = 1 - \left(1 + \frac{0,09}{12}\right)^{-n}$ $\left(1 + \frac{0,09}{12}\right)^{-n} = 0,6862265041\dots$ $-n = \log_{\left(1 + \frac{0,09}{12}\right)} 0,6862265041\dots$ $n = 50,394375\dots$ $n = 51$	✓ substitution in correct formula ✓ answer ✓ substitution ✓ simplification ✓ use of logs ✓ answer (6)
		[13]

Mei/Junie 2023

QUESTION 6/VRAAG 6

6.1.1	$A = P(1+i)^n$ $A = 150\,000(1+0,065)^5$ $A = R205\,513$	✓ substitution into the correct formula ✓ answer (2)
6.1.2	$A = P(1-in)$ $A = 150\,000(1-0,09 \times 5)$ $A = 150\,000 - 67\,000$ $A = R82\,500$	✓ substitution into the correct formula ✓ answer (2)
6.1.3	$SF = A - T = 205\,513 - 82\,500$ $= R123\,013$ $F = \frac{x[(1+i)^n - 1]}{i}$ $x = \frac{F \times i}{(1+i)^n - 1}$ $x = \frac{123\,013 \times \frac{0,0785}{12}}{\left[\left(1 + \frac{0,0785}{12}\right)^{59} - 1\right] \left(1 + \frac{0,0785}{12}\right)}$ $= R1\,704,01$	✓ answer ✓ $i = \frac{0,0785}{12}$ ✓ 59 and $\left(1 + \frac{0,0785}{12}\right)$ (A) ✓ answer (A) (4)
6.2	$P = \frac{x[1 - (1+i)^{-n}]}{i}$ $200\,000 = \frac{6\,000 \left[1 - \left(1 + \frac{0,0525}{4}\right)^{-4n}\right]}{\frac{0,0525}{4}}$ $\frac{7}{16} = 1 - \left(1 + \frac{0,0525}{4}\right)^{-4n}$ $\frac{9}{16} = \left(\frac{1621}{1600}\right)^{-4n}$ $-4n = \frac{\log \frac{9}{16}}{\log \left(\frac{1621}{1600}\right)}$ $-4n = -44,1243 \dots$ $n = 11,03 \text{ years}$	✓ substitution into correct formula ✓ simplification ✓ use of logs ✓ $-4n = -44,1243 \dots$ ✓ $n = 11,03 \text{ years}$ (5)
		[13]

VRAAG 7

QUESTION 7

7.1	$A = P(1+i)^n$ $2 = 1\left(1 + \frac{0,085}{4}\right)^{4n}$ $4n = \log\left(1 + \frac{0,085}{4}\right)^2$ $n = 8,24 \text{ years}$	$\left. \begin{array}{l} \checkmark 2 \\ \checkmark \frac{0,085}{4} \end{array} \right\} \text{ In correct formula}$ $\checkmark \text{ use of logs}$ $\checkmark \text{ answer in years}$ (4)
7.2.1	$A = P(1-i)^n$ $180\,000 = 500\,000(1-i)^5$ $\frac{9}{25} = (1-i)^5$ $\sqrt[5]{\frac{9}{25}} = 1-i$ $i = 0,1848068\dots$ $r = 18,48\%$	$\checkmark \text{ subs into correct formula}$ $\checkmark \text{ simplification}$ $\checkmark i = 0,1848\dots$ $\checkmark \text{ answer}$ (4)
7.2.2	$A = P(1+i)^n$ $A = 500\,000(1 + 0,063)^5$ $A = R678\,635,11$	$\checkmark \text{ subs into correct formula}$ $\checkmark \text{ answer}$ (2)
7.2.3	<p>Sinking Fund = $678\,635,11 - 180\,000$ $= R\,498\,635,11$</p> $498\,635,11 = \frac{x \left[\left(1 + \frac{0,1025}{12}\right)^{58} - 1 \right] \left(1 + \frac{0,1025}{12}\right)^3}{\frac{0,1025}{12}}$ $x = R6\,510,36$	$\checkmark \text{ value of sinking fund}$ $\checkmark \frac{0,1025}{12}$ $\checkmark n = 58 \text{ (A)}$ $\checkmark \left(1 + \frac{0,1025}{12}\right)^3$ $\checkmark \text{ answer (A)}$ (5)
		[15]

Statistiek en Regressie

Mei/Junie 2024

QUESTION/VRAAG 1

1.1	$a = -43,72$ $b = 2,36$ $y = -43,72 + 2,36x$	✓ $a = -43,72$ ✓ $b = 2,36$ ✓ equation (3)
1.2	<p style="text-align: center;">Sprei-diagram</p> <p style="text-align: center;">Gewig (in gram)</p> <p style="text-align: center;">Aantal bladsye</p>	✓ any correct two points ✓ straight line joining the points for $x \in [85 ; 160]$ (2)
1.3	$y = -43,72 + 2,36(110)$ $y = 215,88$ OR $y = 215,90$ (calculator) (sakrekenaar)	✓ substitution ✓ answer (2) ✓✓ answer (2)
1.4	$y = -43,72 + 2,36(130)$ $y = 263,08$ Percentage increase in weight $= \frac{263,08 - 215,88}{215,88} \times 100$ Persentasie toename in gewig $= 21,86\%$ OR $y = 263,08$ Persentasie $= \frac{263,08}{215,88} \times 100$ $= 121,86\%$ Persentasie toename in gewig $= 121,86 - 100 = 21,86$	✓ y -value ✓ difference between y-values ✓ +ve answer (3) ✓ y -value ✓ difference between % ✓ +ve answer (3)
		[10]

QUESTION/VRAAG 2

2.1	<table border="1"> <thead> <tr> <th>Afstand (x km)</th><th>Frekwensie</th><th>Kumulatiewe Frekwensie</th></tr> </thead> <tbody> <tr> <td>$0 \leq x < 5$</td><td>3</td><td>3</td></tr> <tr> <td>$5 \leq x < 10$</td><td>7</td><td>10</td></tr> <tr> <td>$10 \leq x < 15$</td><td>20</td><td>30</td></tr> <tr> <td>$15 \leq x < 20$</td><td>12</td><td>42</td></tr> <tr> <td>$20 \leq x < 25$</td><td>5</td><td>47</td></tr> <tr> <td>$25 \leq x < 30$</td><td>3</td><td>50</td></tr> </tbody> </table>	Afstand (x km)	Frekwensie	Kumulatiewe Frekwensie	$0 \leq x < 5$	3	3	$5 \leq x < 10$	7	10	$10 \leq x < 15$	20	30	$15 \leq x < 20$	12	42	$20 \leq x < 25$	5	47	$25 \leq x < 30$	3	50	<p>✓ 10 ✓ all values correct</p> <p>(2)</p>
Afstand (x km)	Frekwensie	Kumulatiewe Frekwensie																					
$0 \leq x < 5$	3	3																					
$5 \leq x < 10$	7	10																					
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$20 \leq x < 25$	5	47																					
$25 \leq x < 30$	3	50																					
2.2	<p style="text-align: center;"><i>Ogive/Ogief</i></p>	<p>✓ grounding</p> <p>✓ plotting a min of 3 points (cf at upper limits)</p> <p>✓ smooth, increasing curve</p> <p>(3)</p>																					
2.3	<p>$Q_3 = 17,8$ $Q_1 = 11$ $IQR = 6,8$</p>	<p>✓ Q_3 (accept between 17-19) and Q_1 (accept between 10-12,5)</p> <p>✓ answer (accept 5-9)</p> <p>(2)</p>																					

2.4	$5 \leq x < 10$	✓ $5 \leq x < 10$ (1)
2.5	Geskatte gemiddelde $= \frac{2,5(3) + 7,5(11) + 12,5(20) + 17,5(8) + 22,5(5) + 27,5(3)}{50}$ $= \frac{675}{50}$ $= 13,5 \text{ km}$	✓ new frequencies ✓ $\sum fx$ ✓ answer (3)
		[11]

Mei/Junie 2023

QUESTION/VRAAG 1

1.1.1	$a = 1730,22$ $b = 13,96$ $\hat{y} = 1730,22 + 13,96x$	✓ $a = 1730,22$ ✓ $b = 13,96$ ✓ equation (3)
1.1.2	$\hat{y} = 1730,22 + 13,96x$ $\hat{y} = 1730,22 + 13,96(28\,500)$ $\hat{y} = \text{R}399\,590,22$ OR/OF $\hat{y} = \text{R}399\,599,64 \text{ (calc)}$	✓ substitution ✓ answer (2) ✓✓ answer (2)
1.1.3	$r = 0,98002 \dots$ $r = 0,98$	✓ answer (1)
1.1.4	There is a very strong positive correlation between the amount spent on advertising and sales. / <i>Daar is 'n baie sterk positiewe korrelasie tussen die bedrag spandeer op advertensie en die verkope.</i>	✓ strong positive (1)
1.2.1	$\bar{x} = \frac{1\,552\,195}{9}$ $\bar{x} = 172\,466,11$	✓ $\bar{x} = \frac{1\,552\,195}{9}$ ✓ answer (2)
1.2.2	$\sigma = 56\,950,09$	✓ answer (1)
1.2.3	$\bar{x} + \sigma$ $= 172\,466,11 + 56\,950,09$ $= 229\,416,20$ 2 years/jaar	✓ $\bar{x} + \sigma$ ✓ answer (2)
		[12]

QUESTION/VRAAG 2

2.1	$35 < x \leq 45$	✓ answer (1)																								
2.2	320 people/mense	✓ answer (1)																								
2.3	<table border="1"> <thead> <tr> <th>AGE</th><th>NUMBER OF PEOPLE</th><th>CUMULATIVE FREQUENCY</th></tr> </thead> <tbody> <tr> <td>$5 < x \leq 15$</td><td>20</td><td>20</td></tr> <tr> <td>$15 < x \leq 25$</td><td>25</td><td>45</td></tr> <tr> <td>$25 < x \leq 35$</td><td>60</td><td>105</td></tr> <tr> <td>$35 < x \leq 45$</td><td>90</td><td>195</td></tr> <tr> <td>$45 < x \leq 55$</td><td>55</td><td>250</td></tr> <tr> <td>$55 < x \leq 65$</td><td>40</td><td>290</td></tr> <tr> <td>$65 < x \leq 75$</td><td>30</td><td>320</td></tr> </tbody> </table>	AGE	NUMBER OF PEOPLE	CUMULATIVE FREQUENCY	$5 < x \leq 15$	20	20	$15 < x \leq 25$	25	45	$25 < x \leq 35$	60	105	$35 < x \leq 45$	90	195	$45 < x \leq 55$	55	250	$55 < x \leq 65$	40	290	$65 < x \leq 75$	30	320	
AGE	NUMBER OF PEOPLE	CUMULATIVE FREQUENCY																								
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	<p style="text-align: center;">OGIVE/OGIEF</p>	<ul style="list-style-type: none"> ✓ cumulative frequency ✓ grounding ✓ plotting at upper limit ✓ shape <p style="text-align: right;">(4)</p>																								
2.4	<p>Median = 41</p> <p>Mediaan</p>	<p>✓✓ answer</p> <p style="text-align: right;">(2)</p>																								
		[8]																								

QUESTION/VRAAG 1

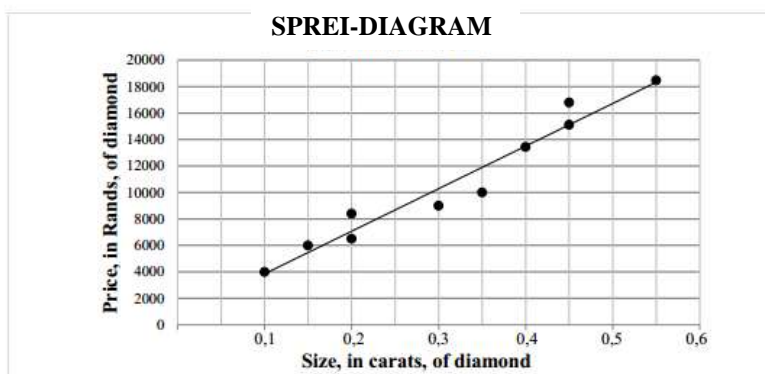
1.1	Modal class: $9 < m \leq 11$ (Modaleklas)	✓ answer (1)																								
1.2	<table border="1"> <thead> <tr> <th>Massa (in kg)</th><th>Frekwensie</th><th>Kumulatiewe Frekwensie</th></tr> </thead> <tbody> <tr> <td>$5 < m \leq 7$</td><td>6</td><td>6</td></tr> <tr> <td>$7 < m \leq 9$</td><td>18</td><td>24</td></tr> <tr> <td>$9 < m \leq 11$</td><td>21</td><td>45</td></tr> <tr> <td>$11 < m \leq 13$</td><td>19</td><td>64</td></tr> <tr> <td>$13 < m \leq 15$</td><td>11</td><td>75</td></tr> <tr> <td>$15 < m \leq 17$</td><td>4</td><td>79</td></tr> <tr> <td>$17 < m \leq 19$</td><td>1</td><td>80</td></tr> </tbody> </table>	Massa (in kg)	Frekwensie	Kumulatiewe Frekwensie	$5 < m \leq 7$	6	6	$7 < m \leq 9$	18	24	$9 < m \leq 11$	21	45	$11 < m \leq 13$	19	64	$13 < m \leq 15$	11	75	$15 < m \leq 17$	4	79	$17 < m \leq 19$	1	80	✓ adding ✓ 80 (2)
Massa (in kg)	Frekwensie	Kumulatiewe Frekwensie																								
$5 < m \leq 7$	6	6																								
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$13 < m \leq 15$	11	75																								
$15 < m \leq 17$	4	79																								
$17 < m \leq 19$	1	80																								
1.3		✓ grounding (5 ; 0) ✓ points ✓ shape (3)																								
1.4	Median mass: 10,5 kg	✓✓ answer (2)																								
1.5.1	$\bar{x} = \frac{(6 \times 6 + 18 \times 8 + 21 \times 10 + 19 \times 12 + 11 \times 14 + 4 \times 16 + 1 \times 18)}{80}$ $= \frac{854}{80}$ $= 10,68$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">Answer only 2/2</div>	✓ 854 ✓ answer (2)																								
1.5.2	Leerders se sakke is swaarder as die voorgeskrewe internasionale riglyn. Geskatte/Beraamde gemiddelde = 10,68 kg 10% of 80 kg = 8 kg 10,68 kg > 8 kg	✓ answer ✓ 8 kg (2)																								

OF

<p>Leerders se sakke is swaarder as die voorgeskrewe internasionale riglyn. Geskatte/Beraamde gemiddelde =</p> $\text{Estimated mean} = \frac{10,68}{80} \times 100$ $= 13,35\%$ <p>13,35% > 10%</p>	<p>✓ answer</p> <p>✓ 13,35%</p> <p>(2)</p>
[12]	

QUESTION/VRAAG 2

Size, in carats, of diamond (x)	0,1	0,15	0,2	0,2	0,3	0,35	0,4	0,45	0,45	0,55
Price, in rands, of diamond (y)	4 000	6 000	6 500	8 400	9 000	10 000	13 440	15 120	16 800	18 480



2.1	$a = 634,382...$ $b = 32\,189,263...$ $\hat{y} = 634,38 + 32189,26x$	<p>Answer only 3/3</p> <p>✓ a ✓ b ✓ equation</p>	(3)
2.2	$\hat{y} = 634,38 + 32189,26(0,25)$ $= R8\,681,70$ OR/OF $\hat{y} = R8\,681,70$ (if using calculator)	<p>✓ substitution ✓ answer</p> <p>✓ ✓ answer</p>	(2) (2)

Gemiddelde Prysverhoging

2.3	<p>Average price increase = $R \frac{32189,26}{20}$ per 0,05 carat $= R1\,609,46$ per 0,05 carat</p> <p>OR/OF</p> <p>Average price increase = $0,05 \times 32189,26$ $= R1\,609,46$ per 0,05 carat</p> <p>OR/OF</p> <p>at 0,3: $\hat{y} = R10\,291,16$ \therefore Average price increase = $10\,291,16 - 8\,681,70$ $= R1\,609,46$ per 0,05 carat</p> <p>Answer only 2/2</p>	<p>✓ divide gradient by 20 ✓ answer</p> <p>✓ multiply gradient by 0,05 ✓ answer</p> <p>✓ Estimated price of a 0,3 carat diamond ✓ answer</p>	(2) (2) (2)
2.4	The point (0,35 ; 11500) is closer to the least squares regression line.	✓ reason	(1)

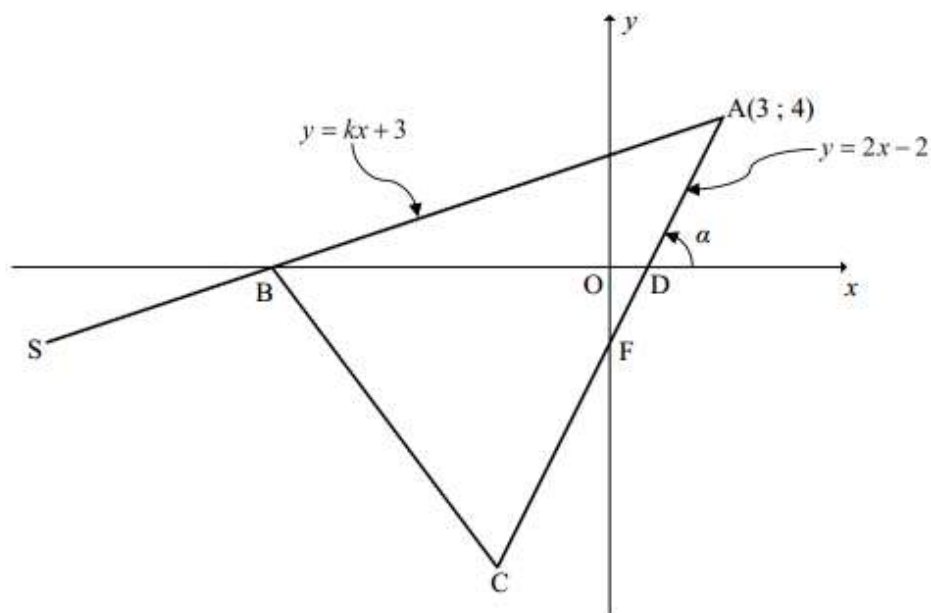
[8]

Die punt (0,35 ; 11500) is nader aan die kleinste-kwadrates regressielyn

Analitiese Meetkunde

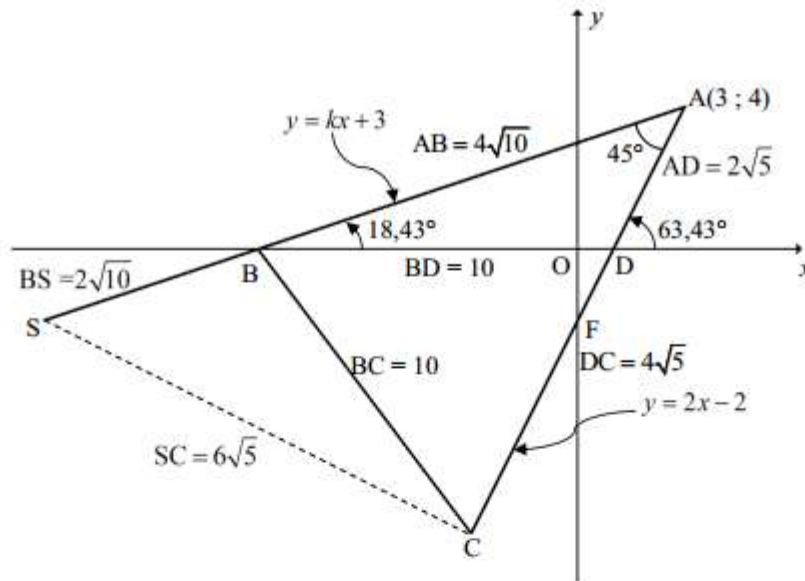
Mei/Junie 2024

QUESTION/VRAAG 3



3.1	$y = kx + 3$ $4 = k(3) + 3$ $3k = 1$ $\therefore k = \frac{1}{3}$ OR y-afsnit van AB: (0 ; 3) $m_{AB} = \frac{4-3}{3-0}$ $= \frac{1}{3}$ $\therefore k = \frac{1}{3}$	✓ substitution (3 ; 4) ✓ substitution (0 ; 3)	(1) (1)
3.2	$0 = \frac{1}{3}x + 3$ $-3 = \frac{1}{3}x$ $x = -9$ B(-9 ; 0)	✓ $y = 0$ ✓ answer	 (2)

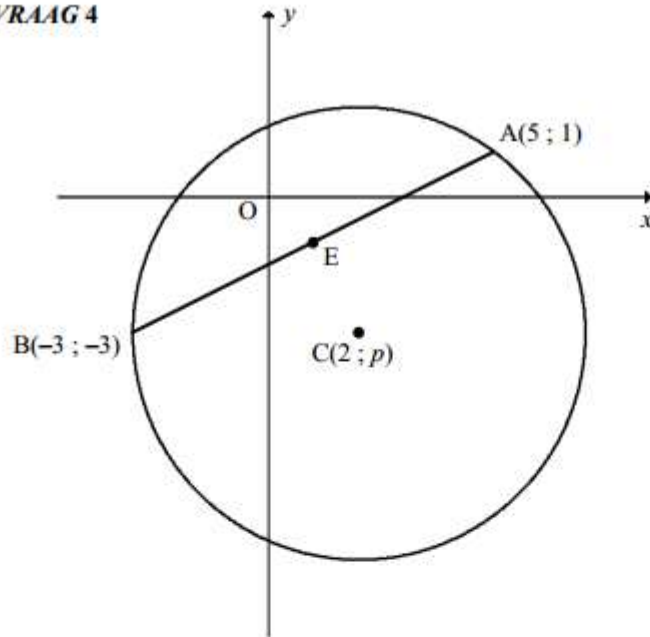
3.3	$F(0; -2)$ $F\left(\frac{x+3}{2}; \frac{y+4}{2}\right)$ $\frac{x+3}{2} = 0 \quad \frac{y+4}{2} = -2$ $x = -3 \quad y = -8$ $C(-3; -8)$ OR by translation OF deur translasie $F(0; -2)$ $A \rightarrow F(x; y) \rightarrow (x-3; y-6)$ $F \rightarrow C(0; -2) \rightarrow (0-3; -2-6) = (-3; -8)$	$\checkmark F(0; -2)$ $\checkmark \frac{x+3}{2} = 0; \frac{y+4}{2} = -2$ $\checkmark x\text{-value} \quad \checkmark y\text{-value}$ (4) $\checkmark F(0; -2)$ $\checkmark (x-3; y-6)$ $\checkmark x\text{-value} \quad \checkmark y\text{-value}$ (4)
3.4	$m_{BC} = \frac{0 - (-8)}{-9 - (-3)} \quad \text{OR} \quad m_{BC} = \frac{-8 - 0}{-3 - (-9)}$ $m_{BC} = -\frac{4}{3}$ $y = -\frac{4}{3}x + c$ $(-2) = -\frac{4}{3}(-15) + c$ $c = -22$ $y = -\frac{4}{3}x - 22$ OR $m_{BC} = \frac{0 - (-8)}{-9 - (-3)} \quad \text{OR} \quad m_{BC} = \frac{-8 - 0}{-3 - (-9)}$ $m_{BC} = -\frac{4}{3}$ $y - y_1 = -\frac{4}{3}(x - x_1)$ $y - (-2) = -\frac{4}{3}(x - (-15))$ $y + 2 = -\frac{4}{3}x - 20$ $y = -\frac{4}{3}x - 22$	\checkmark substitution of B and C into the gradient formula $\checkmark m_{BC}$ $\checkmark m_{\text{line}} = m_{BC}$ \checkmark substitution of S(-15; -2) \checkmark equation (5) \checkmark substitution into the gradient formula $\checkmark m_{BC}$ $\checkmark m_{\text{line}} = m_{BC}$ \checkmark substitution of S(-15; -2) \checkmark equation (5)



3.5	$\tan \alpha = m_{AC} = 2$ $\alpha = 63,43^\circ$ $\tan \hat{A}BD = m_{AS} = \frac{1}{3}$ $\hat{A}BD = 18,43^\circ$ $\hat{B}AC = \alpha - \hat{A}BD$ $\hat{B}AC = 63,43^\circ - 18,43^\circ$ $\hat{B}AC = 45^\circ$ OR $AB = \sqrt{(-9-3)^2 + (0-4)^2}$ $AB = 4\sqrt{10}$ $BD = 10$ $AD = \sqrt{(3-1)^2 + (4-0)^2}$ $AD = 2\sqrt{5}$ $BD^2 = AB^2 + AD^2 - 2AB \cdot AD \cos \hat{B}AC$ $(10)^2 = (4\sqrt{10})^2 + (2\sqrt{5})^2 - 2(4\sqrt{10})(2\sqrt{5}) \cos \hat{B}AC$ $\cos \hat{B}AC = \frac{\sqrt{2}}{2}$ $\hat{B}AC = 45^\circ$	$\checkmark \tan \alpha = m_{AC} = 2$ $\checkmark \alpha = 63,43^\circ$ $\checkmark \tan \hat{A}BD = m_{AS} = \frac{1}{3}$ $\checkmark \hat{A}BD = 18,43^\circ$ \checkmark answer \checkmark length of AB \checkmark calculation of remaining 2 lengths \checkmark substitution into cosine-rule \checkmark rewriting in terms of $\cos \hat{B}AC$ \checkmark answer
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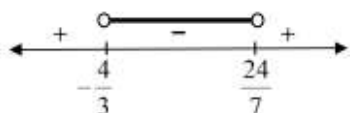
(5)

QUESTION/VRAAG 4



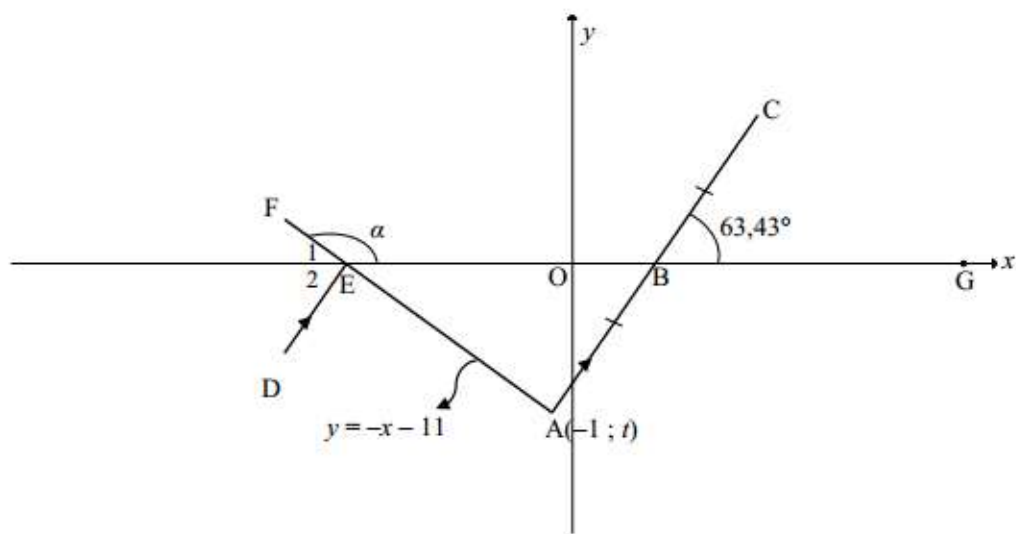
4.1	$E\left(\frac{5+(-3)}{2}; \frac{1+(-3)}{2}\right)$ $\therefore E(1; -1)$	$\checkmark x=1 \quad \checkmark y=-1$ (2)
4.2	$AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$ $AB = \sqrt{(5 - (-3))^2 + (1 - (-3))^2}$ $AB = \sqrt{80} = 4\sqrt{5} = 8,94 \text{ units}$	$\checkmark AB = \sqrt{80} = 4\sqrt{5} = 8,94$ (1)
4.3	$m_{AB} = \frac{1 - (-3)}{5 - (-3)}$ $m_{AB} = \frac{1}{2}$ $\therefore m_{CE} = -2 \quad [CE \perp AB]$ $E(1; -1)$ $y = -2x + c \quad \text{OR} \quad y - y_1 = -2(x - x_1)$ $(-1) = -2(1) + c \quad y - (-1) = -2(x - 1)$ $c = 1 \quad y = -2x + 1$	$\checkmark m_{AB} = \frac{1}{2}$ $\checkmark m_{CE}$ \checkmark substitution of E \checkmark equation (4)

4.4	$y = -2x + 1$ $p = -2(2) + 1$ $p = -3$ OR $m_{CE} = -2$ $\frac{p - (-1)}{2 - 1} = -2$ $p + 1 = -2$ $p = -3$	✓ substitution of $C(2; p)$ into \perp bisector of AB ✓ substitution of C and E into the gradient formula
4.5	$BC = r = 5$ units $\therefore (x - 2)^2 + (y + 3)^2 = 25$ $x^2 - 4x + 4 + y^2 + 6y + 9 = 25$ $x^2 + y^2 - 4x + 6y - 12 = 0$	✓ $BC = r = 5$ units ✓ $(x - 2)^2 + (y + 3)^2 = r^2$ ✓ $x^2 - 4x + 4 + y^2 + 6y + 9 = 25$

4.6	$(x - 2)^2 + (y + 3)^2 = 25$ $y = tx + 8$ $(x - 2)^2 + (tx + 8 + 3)^2 = 25$ $x^2 - 4x + 4 + t^2x^2 + 22tx + 121 - 25 = 0$ $x^2(t^2 + 1) + x(22t - 4) + 100 = 0$ $\Delta < 0$ $(22t - 4)^2 - 4(t^2 + 1)(100) < 0$ $484t^2 - 176t + 16 - 400t^2 - 400 < 0$ $84t^2 - 176t - 384 < 0$ $21t^2 - 44t - 96 < 0$ $(7t - 24)(3t + 4) < 0$ CV: $\frac{24}{7}; -\frac{4}{3}$  $\therefore t \in \left(-\frac{4}{3}; \frac{24}{7}\right)$ OR $-\frac{4}{3} < t < \frac{24}{7}$	✓ substitution of $y = tx + 8$ ✓ standard form ✓ $\Delta < 0$ ✓ standard form of Δ ✓ critical values ✓ answer
		(6)

[18]

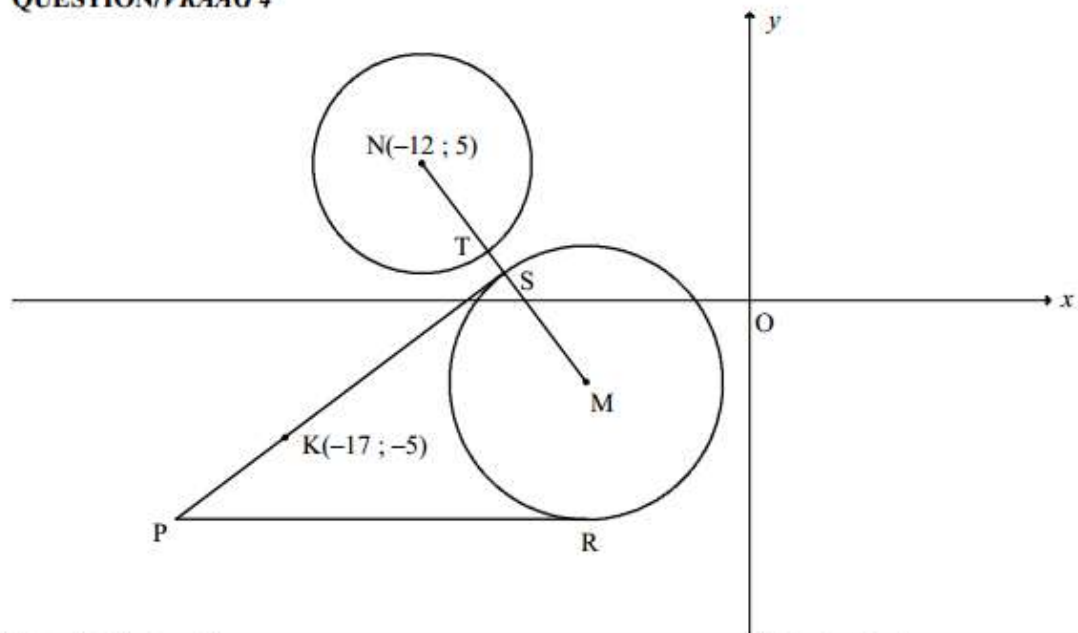
QUESTION/VRAAG 3



3.1.1	$y = -x - 11$ $A(-1; t)$ $t = -(-1) - 11$ $t = -10$	✓ substitution ✓ value of t (2)
3.1.2	$\tan \alpha = -1$ $\text{ref. } \angle = 45^\circ$ $\therefore \alpha = 135^\circ$	✓ $\tan \alpha = -1$ ✓ 135° (2)
3.1.3	$\tan 63,43^\circ = m_{AC}$ $m_{AC} = 2$	✓ $\tan 63,43^\circ = m_{AC}$ ✓ answer (2)
3.2	$m_{AC} = 2$ $A(-1; -10)$ $y = 2x + k$ $-10 = 2(-1) + k$ $k = -8$ $y = 2x - 8$	<p>OR/OF $y - y_1 = 2(x - x_1)$ $y - (-10) = 2(x - (-1))$ $y = 2x - 8$</p> ✓ substitution of m and A ✓ equation (2)

3.3.1	$y = 2x - 8$ $0 = 2x - 8$ $x_B = 4$ $\frac{x_C + (-1)}{2} = 4$ $x_C = 9$ $\frac{y_C + (-10)}{2} = 0$ $y_C = 10$ OR/OF by translation / <i>met translasie</i> $A \rightarrow B(x; y) \rightarrow (x + 5; y + 10)$ $B \rightarrow C(4; 0) \rightarrow (4 + 5; 0 + 10) = (9; 10)$	$\checkmark x_B = 4$ $\checkmark x_C = 9 \quad \checkmark y_C = 10$ (3) $\checkmark (x + 5; y + 10)$ $\checkmark x_C = 9 \quad \checkmark y_C = 10$ (3)
3.3.2	$\hat{A}BE = 63,43^\circ$ $\hat{E}_2 = 63,43^\circ$ $\hat{E}_1 = 45^\circ$ $\hat{F}ED = 108,43^\circ$ OR/OF $\hat{E}AB = 135^\circ - 63,43^\circ$ $\hat{E}AB = 71,57^\circ$ $\hat{D}EA = \hat{E}AB = 71,57^\circ$ $\hat{F}ED = 108,43^\circ$ OR/OF $\hat{A}BE = 63,43^\circ$ $\hat{D}EO = 116,57^\circ$ $\hat{F}ED = 360^\circ - (116,57^\circ + 135^\circ)$ $= 108,43^\circ$	[vert. opp \angle 's =] [corres. \angle 's, DE \parallel AB] [\angle s on a str line] $\checkmark \hat{A}BE = 63,43^\circ$ $\checkmark \hat{E}_1 = 45^\circ$ $\checkmark \hat{F}ED = 108,43^\circ$ (3) $\checkmark \hat{E}AB = 71,57^\circ$ $\checkmark \hat{D}EA = \hat{E}AB = 71,57^\circ$ $\checkmark \hat{F}ED = 108,43^\circ$ (3) [vert. opp \angle 's] [co-int. \angle 's, DE \parallel AB] $\checkmark \hat{A}BE = 63,43^\circ$ $\checkmark \hat{D}EO = 116,57^\circ$ $\checkmark \hat{F}ED = 108,43^\circ$ (3)
3.4	$y = 0$ $x_E = -11$ $\frac{x_G + (-11)}{2} = 4$ $x_G = 19$ $(x - 19)^2 + y^2 = 15^2$ $(x - 19)^2 + y^2 = 225$	$\checkmark x_E = -11$ $\checkmark x_G = 19$ $\checkmark (x - 19)^2 + y^2 = 225$ (4)

QUESTION/VRAAG 4



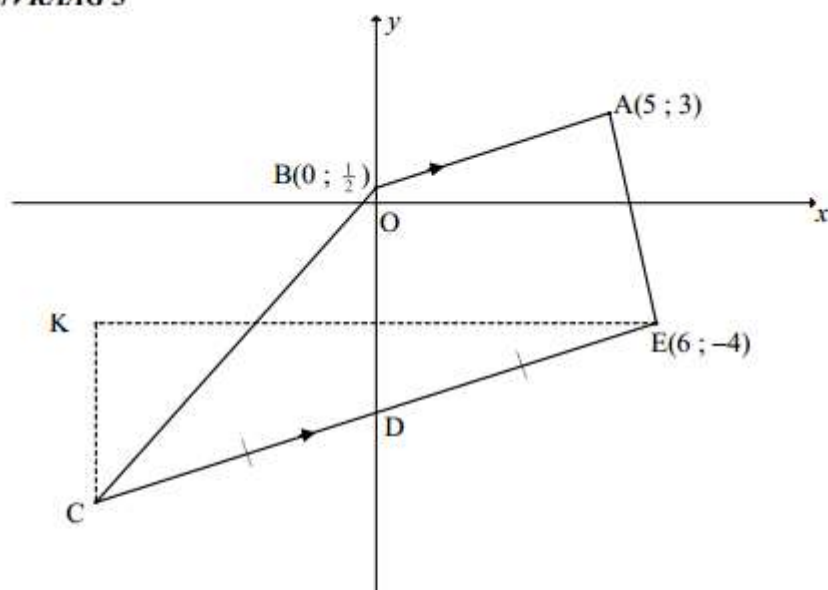
4.1	$M(-6; -3)$	✓ -6 ✓ -3 (2)
4.2.1	$x^2 + y^2 + 24x - 10y + 153 = 0$ $(x+12)^2 + (y-5)^2 = -153 + 144 + 25$ $(x+12)^2 + (y-5)^2 = 16$ $r^2 = 16$ $r = 4$ units	✓ $r^2 = -153 + 144 + 25$ ✓ length of radius (2)
4.2.2	$NM = \sqrt{(-12 - (-6))^2 + (5 - (-3))^2}$ $NM = 10$ units $SM = 5$ units $\therefore TS = 10 - 5 - 4 = 1$ unit	✓ substitution into distance formula ✓ $NM = 10$ units ✓ $SM = 5$ units ✓ answer (4)
4.3.1	$R(-6; -8)$ $y = -8$	✓ $y_R = -8$ ✓ answer (2)

4.3.2	$m_{NM} = \frac{5 - (-3)}{-12 - (-6)}$ $m_{NM} = -\frac{4}{3}$ $m_{\text{tangent}} = \frac{3}{4}$ $-5 = \frac{3}{4}(-17) + c \quad \text{OR/OF} \quad y - y_1 = \frac{3}{4}(x - x_1)$ $c = \frac{31}{4} \quad y - (-5) = \frac{3}{4}(x - (-17))$ $y = \frac{3}{4}x + \frac{31}{4} \quad y = \frac{3}{4}x + \frac{31}{4}$ <p>OR/OF</p> $NS = SM = 5$ $S\left(\frac{-12-6}{2}; \frac{5-3}{2}\right)$ $S(-9; 1)$ $m_{SK} = \frac{1 - (-5)}{-9 + 17}$ $= \frac{6}{8} = \frac{3}{4}$ $y + 5 = \frac{3}{4}(x + 17)$ $y = \frac{3}{4}x + \frac{31}{4} \quad \text{or} \quad y = \frac{3}{4}x + 7\frac{3}{4}$	<p>✓ substitution</p> <p>✓ $m_{NM} = -\frac{4}{3}$</p> <p>✓ $m_{\text{tangent}} = \frac{3}{4}$</p> <p>✓ substitution of m and N</p> <p>✓ equation (5)</p> <p>✓ S midpoint</p> <p>✓ coordinates of S</p> <p>✓ $m_{\text{tangent}} = \frac{3}{4}$</p> <p>✓ substitution of m and $K(-17; -5)$ or S</p> <p>✓ equation (5)</p>
4.4.1	$-8 = \frac{3}{4}x + \frac{31}{4}$ $-32 = 3x + 31$ $3x = -63$ $x = -21$ $P(-21; -8)$ $R(-6; -8) \quad (\text{raaklyne vanaf dieselfde punt})$ <p>$PR = PS = 15$ units [tangents from same point]</p> <p>$MS = MR = 5$ units</p> <p>Omtrek $PSMR = 15 + 15 + 5 + 5$ $= 40$ units</p>	<p>✓ $-8 = \frac{3}{4}x + \frac{31}{4}$</p> <p>✓ $x = -21$</p> <p>✓ $PR = PS = 15$ units</p> <p>✓ $MS = MR = 5$ units</p> <p>✓ answer (5)</p>

4.4.2	$\frac{\text{Oppervlakte van } \triangle NPS}{\text{Opp. van vierhoek PSMR}}$ $\frac{\frac{1}{2} NS.SP}{\frac{1}{2} SP.MS + \frac{1}{2} MR.PR}$ $= \frac{\frac{1}{2}(5)(15)}{2\left(\frac{1}{2}\right)(5)(15)}$ $= \frac{1}{2}$ <p>OR</p> $\frac{\text{area of } \triangle NPS}{\text{area of quadrilateral PSMR}} = \frac{\text{Oppervlakte van } \triangle NPS}{\text{Opp. van vierhoek PSMR}}$ $= \frac{1}{2}$	<p>✓ substitution</p> <p>✓ answer (2)</p> <p>✓ congruent</p> <p>answer (2)</p>
[22]		

Mei/Junie 2022

QUESTION/VRAAG 3



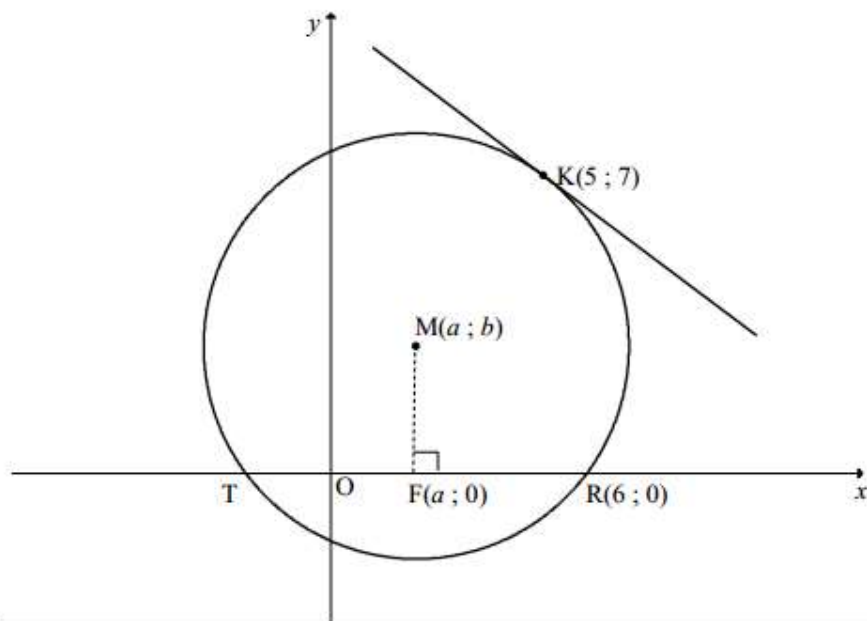
3.1	$m_{AB} = \frac{3 - \frac{1}{2}}{5 - 0}$ $m_{AB} = \frac{1}{2}$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">Answer only 2/2</div>	<p>✓ substitution</p> <p>✓ answer (2)</p>
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3.2	$m_{CE} = m_{BA} = \frac{1}{2}$ $-4 = \frac{1}{2}(6) + c$ OR/OF $y - (-4) = \frac{1}{2}(x - 6)$ $c = -7$ $y = \frac{1}{2}x - 7$	✓ gradient ✓ substitution of E ✓ answer (3)
3.3.1	D(0 ; -7) $\frac{x_c + 6}{2} = 0$ $\frac{y_c + (-4)}{2} = -7$ $x_c = -6$ $y_c = -10$ C(-6 ; -10) <div style="border: 1px solid black; padding: 2px; display: inline-block;">Answer only 3/3</div>	✓ D(0 ; -7) ✓ $x_c = -6$ ✓ $y_c = -10$ (3)
3.3.2	Area $\triangle BCD = \frac{1}{2}(7,5)(6)$ $= 22,5$ Area $\triangle ABD = \frac{1}{2}(7,5)(5)$ $= 18,75$ Area ABCD = $22,5 + 18,75 = 41,25 \text{ units}^2$	✓ subst of correct base and height into the area formula ✓ area $\triangle BCD = 22,5$ ✓ area $\triangle ABD = 18,75$ ✓ answer (4)

Oppervlakte

3.4.1	$K(-6; -4)$	✓ $x_K = -6$ ✓ $y_K = -4$ (2)
3.4.2a	<p>KC = 6 units; KE = 12 units;</p> $CE = \sqrt{(6)^2 + (12)^2} \quad [\text{Pythagoras}]$ $CE = \sqrt{180} = 6\sqrt{5} = 13,42$ <p>Omtrek $\Delta KEC = 6 + 12 + \sqrt{180}$ = 31,42 units</p>	<p>✓ KC = 6 units ✓ KE = 12 units</p> <p>✓ CE</p> <p>✓ answer</p> <p>(4)</p>
3.4.2b	<p>$\tan \hat{KCE} = \frac{KE}{KC} = \frac{12}{6} = 2$ $\hat{KCE} = 63,43^\circ$</p> <p>OR/OF</p> <p>$\sin \hat{KCE} = \frac{KE}{CE} = \frac{12}{\sqrt{180}} = \frac{2\sqrt{5}}{5}$ $\hat{KCE} = 63,43^\circ$</p> <p>OR/OF</p> <p>$m_{CE} = \frac{1}{2}$ $\tan \theta = \frac{1}{2}$ $\theta = 26,57^\circ$ $\hat{KCE} = 90^\circ - 26,57^\circ$ $\hat{KCE} = 63,43^\circ$</p> <p>OR/OF</p> <p>$KE^2 = KC^2 + CE^2 - 2(KC)(CE)\cos \hat{KCE}$ $(12)^2 = (6)^2 + (\sqrt{180})^2 - 2(6)(\sqrt{180})(\cos \hat{KCE})$ $\cos \hat{KCE} = \frac{\sqrt{5}}{5}$ $\hat{KCE} = 63,43^\circ$</p>	<p>✓ trig ratio ✓ $\tan \hat{KCE} = 2$ ✓ answer (3)</p> <p>✓ trig ratio ✓ $\sin \hat{KCE} = \frac{12}{\sqrt{180}}$ ✓ answer (3)</p> <p>✓ $\tan \theta = \frac{1}{2}$ ✓ $\theta = 26,57^\circ$</p> <p>✓ answer (3)</p> <p>✓ substitution into cosine rule ✓ trig ratio ✓ answer (3)</p>

QUESTION/VRAAG 4



4.1.1	$y = x + 1$ $b = a + 1$	$\checkmark b = a + 1$ (1)
4.1.2	$MR^2 = MK^2$ $(a - 6)^2 + (b - 0)^2 = (a - 5)^2 + (b - 7)^2$ $(a - 6)^2 + (a + 1)^2 = (a - 5)^2 + (a + 1 - 7)^2$ $a^2 + 2a + 1 = a^2 - 10a + 25$ $12a = 24$ $a = 2$ $b = 3$ $\therefore M(2; 3)$	\checkmark equating radii / solving simultaneously \checkmark substitution $b = a + 1$ $\checkmark 12a = 24$ $\checkmark a = 2$ $\checkmark b = 3$ (5)
4.2.1	$(6 - 2)^2 + (0 - 3)^2 = r^2$ $r = 5$ OR/OF $(2 - 5)^2 + (3 - 7)^2 = r^2$ $r = 5$	\checkmark substitution R and M $\checkmark r = 5$ (2) \checkmark substitution K and M $\checkmark r = 5$ (2)

Answer only 2/2

4.2.2	<p>T(-2 ; 0) TR = 8 units [line from centre \perp to chord] [lyn vanaf middelpunt \perp op koord]</p> <p>OR/OF</p> <p>M(2 ; 3) F(a ; 0) FR = 4 units TR = 8 units [line from centre \perp to chord] [lyn vanaf middelpunt \perp op koord]</p> <p>OR/OF</p> <p>$(x-2)^2 + (0-3)^2 = 25$ $x^2 - 4x + 4 + 9 = 25$ $x^2 - 4x - 12 = 0$ $(x-6)(x+2) = 0$ $x = 6$ or $x = -2$ TR = 8 units</p>	<p>✓ T(-2 ; 0) ✓ answer (2)</p> <p>✓ 4 units ✓ answer (2)</p> <p>✓ x values ✓ answer (2)</p>
4.3	<p>$m_{\text{radius}} = \frac{7-3}{5-2}$ $m_{\text{radius}} = \frac{4}{3}$ $m_{\text{tangent}} = -\frac{3}{4}$</p> <p>$7 = -\frac{3}{4}(5) + c$ OR/OF $y - 7 = -\frac{3}{4}(x - 5)$ $c = \frac{43}{4}$ $y = -\frac{3}{4}x + \frac{43}{4}$</p>	<p>✓ substitution ✓ $m_{\text{radius}} = \frac{4}{3}$ ✓ $m_{\text{tangent}} = -\frac{3}{4}$ ✓ substitution ✓ answer (5)</p>
4.4.1	N(2 ; -2)	<p>✓ $x_N = 2$ ✓ $y_N = -2$ (2)</p>
4.4.2	<p>$(-2-2)^2 + (0+2)^2 = r^2$ $r^2 = 20$ $(x-2)^2 + (y+2)^2 = 20$</p>	<p>✓ substitution ✓ $r^2 = 20$ ✓ answer (3)</p>
		[20]

